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GAUGE COUPLING CONSTANT UNIFICATION WITH PLANCK SCALE VALUES OF MODULI

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ABSTRACT

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Convergence of the standard model gauge coupling constants to a common value at around 2×10^{16} GeV is studied in the context of orbifold theories where the modular symmetry groups for T and U moduli are broken to subgroups of $PSL(2, Z)$. The values of the moduli required for this unification of coupling constants are studied for this case and also for the case where string unification is accompanied by unification to a gauge group larger then $SU(3) \times SU(2) \times U(1)$.

When the standard model gauge coupling constants are extrapolated [1, 2] to high energies using the renormalization group equations of the minimal supersymmetric model with just two Higgs doublets the three gauge couplings g_3, g_2 and g_1 of $SU(3) \times SU(2) \times U(1)$ attain a common value at about 2×10^{16} GeV. There is a problem in obtaining consistency with heterotic string theory because tree level gauge coupling constants in the string theory have a common value [3] at a string unification scale M_{string} around 0.37×10^{18} GeV. Amongst the possible ways of arranging consistency are grand unification of the gauge group to $SU(5)$ or $SO(10)$ at 2×10^{16} GeV with the coupling constants then running with a common value to M_{string}, modification of the running of the renormalization group equations by the inclusion of extra states [4-8] with mass intermediate between the electroweak scale and the string unification scale, and, in the context of orbifold compactification, inclusion of moduli dependent string loop threshold corrections [9-12] in the renormalization group equations for the standard model or for models with $SU(5) \times U(1)$, $SO(4) \times SO(6)$ or $[SU(3)]^3$ unification [13]. Unification of gauge coupling constants has also been studied in the context of free fermion models [8]. Furthermore, an alternative approach in which one considers non-standard values of the Kac-Moody levels within the minimal supersymmetric standard model has been studied in [14].

The first of these approaches requires the gauge group of the heterotic string theory to be at least at level two to permit Higgs scalars in the adjoint representation [15]. It has proved difficult to construct realistic models of this type despite considerable efforts [16]. The second approach requires us to believe that the observed unification of gauge coupling constants at 2×10^{16} GeV using supersymmetric standard model renormalization group equations is a coincidence, and without unification to a gauge group larger than $SU(3) \times SU(2) \times U(1)$, the third approach appears to require large values of the orbifold moduli to give a sufficiently large threshold corrections [17,18]. However, it has been assumed in the latter calculations that the threshold corrections are those with $PSL(2, Z)$ modular symmetry in the T and U moduli. These modular symmetry groups can be

broken [19, 20] to subgroups of $PSL(2,Z)$ when the orbifold lattice is such that some twisted sectors have fixed planes for which the six-torus $\mathbf{T}_6$ cannot be decomposed into a direct sum $\mathbf{T}_2 \oplus \mathbf{T}_4$ with the fixed plane lying in $\mathbf{T}_2$. We shall refer to this as the non $\mathbf{T}_2 \oplus \mathbf{T}_4$ case. The modified form of the threshold corrections is known [19, 20]. Modular symmetries of threshold corrections may also be broken by discrete Wilson line backgrounds [18, 21, 22] though in this case it has not been possible to date to calculate the form of the threshold corrections.

We shall investigate the effect of threshold corrections with broken modular symmetries on the values of the T and U moduli for which unification of gauge group couplings constants occurs at 2×10^{16}. (In a recent paper [23] it has been shown that the inclusion of Wilson line moduli along with T and U moduli can result in substantially smaller values of moduli being needed.) We shall also study the values of the moduli required to achieve this unification of gauge coupling constants when the gauge group above the unification scale is larger than $SU(3) \times SU(2) \times U(1)$.

In general, the renormalization group equations, including string loop threshold corrections, for a semi-simple gauge group with factors G_a, all at level 1, may be written in the form

$$16\pi^2 g_a^{-2}(\mu) = 16\pi^2 g_{string}^{-2} + b_a log\Big(\frac{M_{string}^2}{\mu^2}\Big) + \Delta_a, \tag{1}$$

where g_{string} is the common value of the gauge coupling constants at the string tree level unification scale M_{string} with approximate values

$$M_{string} \approx 0.527 g_{string} \times 10^{18}\text{GeV} \tag{2}$$

and

$$g_{string} \approx 0.7 \tag{3}$$

In the non $\mathbf{T}_2 \oplus \mathbf{T}_4$ cases, with modular symmetries that are subgroups of

$PSL(2,Z)$, the threshold corrections take the form [19, 20]

$$\begin{aligned}\Delta_a = & -\sum_i ({b'}_a^{\,i} - \delta^i_{GS})\Big(ln(T_i + \bar{T}_i) + \sum_m \frac{C_{im}}{2} ln|\eta(\frac{T_i}{l_{im}})|^4 \Big) \\ & - \sum_i ({d'}_a^{\,i} - \tilde{\delta}^i_{GS})\Big(ln(U_i + \bar{U}_i) + \sum_m \frac{\tilde{C}_{im}}{2} ln|\eta(\frac{U_i}{\tilde{l}_{im}})|^4 \Big)\end{aligned} \tag{4}$$

where the sum over i is restricted to $N = 2$ complex planes, which are unrotated in at least one twisted sector, and for the U moduli is further restricted to complex planes for which the point group acts as Z_2. The range over which m runs depends on the value of i, but

$$\sum_m \tilde{C}_{im} = \sum_m C_{im} = 2, \qquad \forall i, \tag{5}$$

The coefficients C_{im}, l_{im}, $\tilde{C}_{im}$ and $\tilde{l}_{im}$ are given in [24] for the various non $\mathbf{T}_2 \oplus \mathbf{T}_4$ Coxeter $\mathbf{Z}_N$ orbifolds. In the case of the $\mathbf{Z}_6 - II - b$ orbifold, the modulus U_3 is understood to be replaced by $U_3 + 2i$ in the argument of the Dedekind eta function. The quantities δ^i_{GS} and $\tilde{\delta}^i_{GS}$ are the Green-Schwarz coefficients, and the coefficients ${b'}_a^{\,i}$ and ${d'}_a^{\,i}$, which are determined by the contribution of the massless states to the modular anomaly [12] in a way that does not depend on the underlying lattice of the orbifold are given by

$${b'}_a^{\,i} = -C(G_a) + \sum_{R_a} T(R_a)(1 + 2n^i_{R_a}) \tag{6}$$

and

$${d'}_a^{\,i} = -C(G_a) + \sum_{R_a} T(R_a)(1 + 2l^i_{R_a}) \tag{7}$$

where $C(G_a)$ and $T(R_a)$ are Casimirs for the gauge group factor G_a and its representations R_a, and $n^i_{R_a}$ and $l^i_{R_a}$ are the modular weights under T_i and U_i modular transformations, respectively, for massless states in the representation R_a of G_a. All possible values of $n^i_{R_a}$ and $l^i_{R_a}$ have been determined [18, 25] for massless states in arbitrary twisted sectors of abelian Coxeter orbifolds.

If g_a and g_b are the gauge coupling constants for 2 factors of the $SU(3) \times SU(2) \times U(1)$ standard model gauge group, and if the unification scale at which all 3 gauge coupling constants converge to a common value M_X with

$$M_X = 2 \times 10^{16} \tag{8}$$

then, from (1) and (4),

$$\frac{M^2_{string}}{M^2_X} = \prod_i \alpha_i^{\frac{({b'}_a^{\,i} - {b'}_b^{\,i})}{(b_a - b_b)}} \tilde{\alpha}_i^{\frac{({d'}_a^{\,i} - {d'}_b^{\,i})}{(b_a - b_b)}} \tag{9}$$

where

$$\alpha_i = (T_i + \bar{T}_i) \prod_m |\eta(\frac{T_i}{l_{im}})|^{2C_{im}} \tag{10}$$

and

$$\tilde{\alpha}_i = (U_i + \bar{U}_i) \prod_m |\eta(\frac{U_i}{\tilde{l}_{im}})|^{2\tilde{C}_{im}} \tag{11}$$

In the case of α_i, the product in (9) sums over all $N = 2$ complex planes, and in the case of $\tilde{\alpha}_i$ over all $N = 2$ complex planes for which the point group acts as Z_2.

For the supersymmetric standard model with $SU(3) \times SU(2) \times U(1)$ gauge group, 3 generations of quarks and leptons and higgses h and $\bar{h}$, the renormalization group coefficients b_a are

$$b_3 = -3,\ b_2 = 1,\ b_1 = \frac{33}{5}. \tag{12}$$

In terms of the modular weights for the massless matter fields, the coefficients ${b'}_a^{\,i}$

are given by

$$
{b'_3}^i = 3 + \sum_{g=1}^{3} (2n^i_{Q(g)} + n^i_{u(g)} + n^i_{d(g)}) \tag{13}
$$

$$
{b'_2}^i = 5 + n^i_h + n^i_{\bar{h}} + \sum_{g=1}^{3} (3n^i_{Q(g)} + n^i_{L(g)}) \tag{14}
$$

and

$$
{b'_1}^i = \frac{33}{5} + \frac{3}{5}(n^i_h + n^i_{\bar{h}}) + \frac{1}{5}\sum_{g=1}^{3} (n^i_{Q(g)} + 8n^i_{u(g)} + 2n^i_{d(g)} + 3n^i_{L(g)} + 6n^i_{e(g)}) \tag{15}
$$

where g labels the generations, and $L(g)$ and $Q(g)$ are lepton and quark $SU_L(2)$ doublets. Exactly similar expressions apply for ${d'_a}^i$ with n^i replaced by l^i.

For a given twisted sector of a given orbifold the possible modular weights of matter states can be calculated from the twists on the string degrees of freedom and the left mover oscillators involved in the construction of the states [18, 25]. In general, for a massless left mover the oscillator number $\tilde{N}$ is given by

$$
\tilde{N} = a_L - h_{KM} \tag{16}
$$

where a_L is the normal ordering constant for the particular orbifold twisted sector and h_{KM} is the contribution to the conformal weight of the state from the $E_8 \times E_8$ algebra. For level 1 gauge group factors G_a, the contribution is given by [18]

$$
h_{KM} = \sum_a \frac{dimG_a}{dimR_a} \frac{T(R_a)}{C(G_a)+1} \tag{17}
$$

For the $SU(3) \times SU(2) \times U(1)$ case, the relevant conformal weights are

$$
h_{KM} \geq \frac{3}{5}, \quad \text{for } Q, u, e, \tag{18}
$$

and

$$
h_{KM} \geq \frac{2}{5}, \quad \text{for } L, d, h, \bar{h}, \tag{19}
$$

where the inequality allows for any additional contributions to h_{KM} from extra

$U(1)$ factors in the gauge group assumed to be spontaneously broken along flat directions at a high energy scale.

Because the complex planes for which both T and U moduli occur are planes where the point group acts as Z_2, the modular weights associated with the T and U modulus for these planes are the same state by state. As a consequence, for such complex planes we have ${b'_a}^i$ and ${d'_a}^i$ equal. Thus (9) simplifies to

$$\frac{M_{string}^2}{M_X^2} = \prod_j \alpha_j^{\frac{({b'_a}^j - {b'_b}^j)}{(b_a - b_b)}} \prod_k (\alpha_k \tilde{\alpha}_k)^{\frac{({b'_a}^k - {b'_b}^k)}{(b_a - b_b)}} \tag{20}$$

where the product over k is for the $N = 2$ Z_2 planes and the product over j is for all other $N = 2$ complex planes. For all non $\mathbf{T}_2 \oplus \mathbf{T}_4$ $\mathbf{Z}_N$ orbifolds except $\mathbf{Z}_6 - II - a, b, c$, there is only one $N = 2$ complex plane and so only one complex plane contributing to the threshold corrections. Thus, for all except the $\mathbf{Z}_6 - II$ cases, either

$$\frac{M_{string}^2}{M_X^2} = \alpha_3^{\frac{({b'_a}^3 - {b'_b}^3)}{(b_a - b_b)}} \tag{21}$$

where the $N = 2$ complex plane, taken to be the third complex plane, is a plane where the point group acts as Z_M for $M \neq 2$ or

$$\frac{M_{string}^2}{M_X^2} = (\alpha_3 \tilde{\alpha}_3)^{\frac{({b'_a}^3 - {b'_b}^3)}{(b_a - b_b)}} \tag{22}$$

if the $N = 2$ complex plane is a plane where the point group acts as Z_2.

For all 3 gauge coupling constants to converge to a single value at the same scale M_X it is necessary that

$$\frac{(b'^3_3 - b'^3_2)}{(b'^3_3 - b'^3_1)} = \frac{(b_3 - b_2)}{(b_3 - b_1)} = \frac{5}{12} \tag{23}$$

For this energy scale to be less than M_{string} the sign needed for the exponent in (21) or (22) depends on whether α_3 or $\alpha_3 \tilde{\alpha}_3$ is greater than or less than 1. A

numerical study of the variation of the functions α_i and $\tilde{\alpha}_i$ of (10) and (11) with T_i and U_i for the various non $\mathbf{T}_2 \oplus \mathbf{T}_4$ orbifolds listed in [24] shows that, although it is possible for certain of these functions to attain values greater than 1, they are never much greater than 1 (never greater than 1.42) On the other hand, the functions α_i and $\tilde{\alpha}_i$ can take values very much smaller than 1 for sufficiently large values of T_i and U_i. Thus, for the values of the exponent $\frac{({b'_a}^3 - {b'_b}^3)}{(b_a - b_b)}$ obtained in practice (numerically less than one) there is no possibility of obtaining a value of $\frac{M^2_{string}}{M^2_X}$ of the required magnitude except for the case where α_3 or $\alpha_3\tilde{\alpha}_3$ is less than 1 and raised to a negative power. We must therefore require

$$\frac{({b'_3}^3 - {b'_2}^3)}{(b_3 - b_2)} < 0. \tag{24}$$

In addition, cancellation of modular anomalies [12] for the $N = 1$ complex planes requires

$${b'_3}^i = {b'_2}^i = {b'_1}^i, \; i = 1, 2. \tag{25}$$

No further conditions arise from modular anomalies associated with the U moduli because the modular weights for the T and U moduli associated with a complex plane are the same state by state.

The conditions to be satisfied for a solution where all 3 gauge coupling constants converge to a common value at a value of M_X less than M_{string} for the non $\mathbf{T}_2 \oplus \mathbf{T}_4$ examples of the $\mathbf{Z}_4$, $\mathbf{Z}_8 - II$ and $\mathbf{Z}_{12} - I$ orbifolds are now identical to the conditions considered in [18] for the $\mathbf{T}_2 \oplus \mathbf{T}_4$ versions of these orbifolds. The only difference is the values of T_3 (and U_3 which, for simplicity, was not included in [18]) to obtain unification at 2×10^{16} GeV would differ because the functions α_3 and $\alpha_3\tilde{\alpha}_3$ differ from $(T_3 + \bar{T}_3) ln|\eta(T_3)|^4$. Since no solutions were found to exist in the $\mathbf{T}_2 \oplus \mathbf{T}_4$ case, there are still no solutions for these orbifolds.

This leaves only $\mathbf{Z}_6 - II - a, b, c$ as candidate non $\mathbf{T}_2 \oplus \mathbf{T}_4$ $\mathbf{Z}_N$ orbifolds for a successful unification of gauge coupling constants. For these cases, (20) simplifies

to

$$\frac{M^2_{string}}{M^2_X} = \alpha_1^{\frac{(b'_a{}^1 - b'_b{}^1)}{(b_a - b_b)}} (\alpha_3 \tilde{\alpha}_3)^{\frac{(b'_a{}^3 - b'_b{}^3)}{(b_a - b_b)}} \tag{26}$$

where

$$\begin{aligned}
\alpha_1 =& (T_1 + \bar{T}_1)|\eta(\frac{T_1}{2})|^4, \qquad \alpha_3 = (T_3 + \bar{T}_3)|\eta(T_3)|^2|\eta(\frac{T_3}{3})|^2, \\
\tilde{\alpha}_3 =& (U_3 + \bar{U}_3)|\eta(U_3)|^2|\eta(3U_3)|^2, \\
& \text{for } \mathbf{Z}_6 - II - a \\
\\
\alpha_1 =& (T_1 + \bar{T}_1)|\eta(T_1)|^4, \qquad \alpha_3 = (T_3 + \bar{T}_3)|\eta(T_3)|^2|\eta(\frac{T_3}{3})|^2, \\
\tilde{\alpha}_3 =& (U_3 + \bar{U}_3)|\eta(U_3 + 2)|^2|\eta(\frac{U_3 + 2}{3})|^2, \\
& \text{for } \mathbf{Z}_6 - II - b \\
\\
\alpha_1 =& (T_1 + \bar{T}_1)|\eta(T_1)|^4, \qquad \alpha_3 = (T_3 + \bar{T}_3)|\eta(T_3)|^2|\eta(\frac{T_3}{3})|^2, \\
\tilde{\alpha}_3 =& (U_3 + \bar{U}_3)|\eta(U_3)|^2|\eta(3U_3)|^2, \\
& \text{for } \mathbf{Z}_6 - II - c
\end{aligned} \tag{27}$$

Solutions with the 3 gauge coupling constants converging to a single value at the same scale M_X have been found [18] for the $\mathbf{T}_2 \oplus \mathbf{T}_4$ version of the $\mathbf{Z}_6 - II$ orbifold only for the case in which the threshold corrections are dominated by T_1 and do not depend significantly on T_3 and U_3, and, as we have argued above, the conditions to be satisfied for a solution to exist are identical here. Then, it is α_1 that determines the value of $\frac{M^2_{string}}{M^2_X}$ and the value T_1 required for unification at 2×10^{16} GeV is either the same as in the $\mathbf{T}_2 \oplus \mathbf{T}_4$ case or somewhat larger. (Had solutions existed with T_3 and U_3 dominating the threshold corrections, the presence of the factor $|\eta(3U_3)|^2$ in $\tilde{\alpha}_3$ for the $\mathbf{Z}_6 - II - a$ and $\mathbf{Z}_6 - II - c$ cases would have allowed unification with smaller values of the moduli.)

Convergence of the gauge coupling constants to a common value at M_X may perhaps be achieved with smaller values of the moduli when the modular symmetries are broken instead [21, 22] by the presence of discrete Wilson lines. For

example, the choice of Wilson lines given in eqn (60) of the first reference of [21] when applied to the $\mathbf{Z}_3$ plane of the $\mathbf{Z}_6 - II$ orbifold (in the $\mathbf{T}_2 \oplus \mathbf{T}_4$ version) gives modular symmetry group $\Gamma_0(3)$ for T_1. It is not known at this time how to calculate the explicit threshold corrections with discrete Wilson lines. However, if we conjecture a simple form consistent with the modular symmetries by employing Dedekind eta functions as in (4), then we might replace α_1 in (27) by

$$\alpha_1 = (T_1 + \bar{T}_1)|\eta(3T_1)|^4. \tag{28}$$

The orbifold solution with T_1 dominating the threshold corrections will then give convergence of the gauge coupling constants to a common value at $M_X \approx 2 \times 10^{16}$ GeV with

$$ReT_1 \approx 8.3. \tag{29}$$

if we make the choice of modular weights displayed in [18] for which

$$\frac{({b'_2}^1 - {b'_3}^1)}{(b_2 - b_3)} = -\frac{1}{4} \tag{30}$$

This is to be compared with $ReT_1 \approx 26$ when the modular symmetry is unbroken.

Another possible mechanism for convergence of the gauge coupling constants to a common value to occur at 2×10^{16} GeV with moderate values of the moduli is to have the string unification of $SU(3) \times SU(2) \times U(1)$ gauge coupling constants accompanied by unification to a gauge group larger than $SU(3) \times SU(2) \times U(1)$. In an earlier paper [13], it has been shown that such a unification of coupling constants can occur for a number of $\mathbf{Z}_M \times \mathbf{Z}_N$ orbifolds (though not for $\mathbf{Z}_N$ orbifolds) with unified gauge group $[SU(3)]^3$ or $SO(4) \times SO(6)$.

For the case of unification to $[SU(3)]^3$ with the minimal massless matter content [26] of three copies of $(\mathbf{3}, \mathbf{3}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) + (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$ to provide the generations and electroweak Higgses and 2 copies of $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ providing the $[SU(3)]^3$ breaking Higgses H and $\bar{H}$ above the unification scale, and the massless matter

content of the supersymmetric standard model below the unification scale, the difference of the coefficients ${b'_2}^2$ and ${b'_3}^i$ for the $SU(3) \times SU(2) \times U(1)$ threshold corrections is given by

$$ {b'_2}^i - {b'_3}^i = 6 + 3\sum_{g=1}^{3} \Big(n_g^i(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) - n_g^i(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \Big) + 3\sum_{f=1}^{2} \Big(n_f^i(H) + n_f^i(\bar{H}) \Big). \quad (31) $$

For the $[SU(3)]^3$ case, all possible choices of modular weights to satisfy the conditions for the $SU(3) \times SU(2) \times U(1)$ gauge coupling constants to converge to a common value at a scale less than M_{string} with a single T_i modulus dominating the threshold corrections can be generated using eqns (27) and (28) of ref. [13] together with a knowledge of all allowed modular weights of massless states in the twisted sectors of $\mathbf{Z}_M \times \mathbf{Z}_N$ orbifolds [18, 24] when the contribution to the modular weight of the state from the $E_8 \times E_8$ algebra h_{KM} satisfies $h_{KM} \geq \frac{2}{3}$. We have tabulated in table 2 all the possible values of the exponent ρ in

$$ \frac{M_{string}^2}{M_X^2} = \Big((T_d + \bar{T}_d)|\eta(T_d)|^4 \Big)^{\rho} \quad (32) $$

where T_d is the dominant modulus and

$$ \rho = \frac{({b'_2}^d - {b'_3}^d)}{(b_2 - b_3)} \quad (33) $$

We have also tabulated the values of ReT_d which then produce convergence of the gauge coupling constants to a common value at 2×10^{16} GeV. Our notations for the $\mathbf{Z}_M \times \mathbf{Z}_N$ orbifolds are as in table 1. It can be seen that this can be achieved for values of ReT_d as small as 3.8. (The non $\mathbf{T}_2 \oplus \mathbf{T}_4$ case need not be considered here because the only $\mathbf{Z}_M \times \mathbf{Z}_N$ Coxeter orbifold for which there are such lattices [27] is $\mathbf{Z}_2 \times \mathbf{Z}_2$ and there are then no unification solutions [13] in either the $[SU(3)]^3$ or the $SO(4) \times SO(6)$ case.)

For the case of unification to $SO(4)\times SO(6)$ with the minimal massless matter content [28] of three copies of $(\mathbf{2},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{2},\bar{\mathbf{4}})$ to provide the generations and one copy each of $(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{6})$ and $H+\bar{H}=(\mathbf{1},\mathbf{2},\mathbf{4})+(\mathbf{1},\mathbf{2},\bar{\mathbf{4}})$ above the unification scale and the massless matter content of the supersymmetric standard model below the unification scale, we have instead

$$b'{}_2{}^i - b'{}_3{}^i = 2\sum_{g=1}^{3}\Big(n_g^i(\mathbf{2},\mathbf{1},\mathbf{4})-n_g^i(\mathbf{1},\mathbf{2},\bar{\mathbf{4}})\Big)+2\Big(n^i(\mathbf{2},\mathbf{2},\mathbf{1})-(n_H^i+n_{\bar{H}}^i)-n^i(\mathbf{1},\mathbf{1},\mathbf{6})\Big). \tag{34}$$

For the $SO(4)\times SO(6)$ case, the conditions for the $SU(3)\times SU(2)\times U(1)$ gauge coupling constants to converge to a common value at a scale less than M_{string} with a single T_i modulus dominating the threshold corrections are eqns. (23) and (24) of ref. [13], and in this case the allowed modular weights of the twisted sector massless states for $\mathbf{Z}_M\times\mathbf{Z}_N$ orbifolds are those for which h_{KM} satisfies $h_{KM}\geq\frac{5}{8}$ for $(\mathbf{2},\mathbf{1},\mathbf{4}),(\mathbf{1},\mathbf{2},\bar{\mathbf{4}})$ and $(\mathbf{1},\mathbf{2},\mathbf{4})$ and $h_{KM}\geq\frac{1}{2}$ for $(\mathbf{2},\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\mathbf{6})$. We have tabulated in table 3, the range of allowed values of the exponent ρ of (32) together with the values of the T_d for which convergence of the $SU(3)\times SU(2)\times U(1)$ gauge coupling constants to a common value at $2\times 10^{16}GeV$ is achieved. It can be seen that this can be achieved for values of ReT_d as small as 3.5

In conclusion, a study has been made of convergence of gauge coupling constants to a common value at 2×10^{16} GeV in the context of non $\mathbf{T}_2\oplus\mathbf{T}_4$ $\mathbf{Z}_N$ orbifolds where the modular symmetries of threshold corrections are subgroups of $PSL(2,Z)$. The only non $\mathbf{T}_2\oplus\mathbf{T}_4$ orbifolds for which this unification of gauge coupling constants occurs are those for which it already occurred for the $\mathbf{T}_2\oplus\mathbf{T}_4$ version of the orbifold. In no case can the unification at 2×10^{16} GeV be achieved with smaller values of the moduli than in the $\mathbf{T}_2\oplus\mathbf{T}_4$ case. However, when the $PSL(2,Z)$ modular symmetries are broken instead by discrete Wilson lines, smaller values of the moduli may be possible, though there is uncertainty as to the detailed form of the threshold corrections in this case. We have also considered convergence of gauge coupling constants to a common value when string unification is accom-

panied by unification to a gauge group larger than $SU(3)\times SU(2)\times U(1)$ and have found values of the dominant T_i modulus of around 3, in Planck scale units. will allow convergence of $SU(3)\times SU(2)\times U(1)$ gauge coupling constants to occur at 2×10^{16} GeV accompanied by either $[SU(3)]^3$ or $SO(4)\times SO(6)$ unification.

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Table Captions

Table. 1. $\mathbf{Z}_M\times\mathbf{Z}_N$ orbifolds. For the point group generator ω we display $(\zeta_1,\zeta_2,\zeta_3)$ such that the action of w on the complex plane orthogonal basis is $(e^{2\pi i\zeta_1},e^{2\pi i\zeta_2},e^{2\pi i\zeta_3})$ and similarly for the point group generator ϕ.

Table. 2. Values of the exponent ρ in (32) and ReT_d for the various $\mathbf{Z}_M\times\mathbf{Z}_N$ orbifolds for threshold corrections dominated by a single modulus T_d and unification to $[SU(3)]^3$ at 2×10^{16} GeV.

Table. 3. Values of the exponent ρ in (32) and ReT_d for the various $\mathbf{Z}_M\times\mathbf{Z}_N$ orbifolds for threshold corrections dominated by a single modulus T_d and unification to $SO(4)\times SO(6)$ at 2×10^{16} GeV.

Orbifold	Point group generator ω	Point group generator ϕ
$Z_2 \times Z_2$	$(1,1,0)/2$	$(0,1,1)/2$
$Z_4 \times Z_2$	$(1,-1,0)/4$	$(0,1,1)/2$
$Z_6 \times Z_2$	$(1,-1,0)/6$	$(0,1,1)/2$
$Z_6' \times Z_2$	$(1,1,4)/6$	$(0,1,1)/2$
$Z_3 \times Z_3$	$(1,2,0)/3$	$(0,1,2)/3$
$Z_6 \times Z_3$	$(1,5,0)/6$	$(0,1,2)/3$
$Z_4 \times Z_4$	$(1,-1,0)/4$	$(0,1,-1)/4$
$Z_6 \times Z_6$	$(1,5,0)/6$	$(0,1,5)/6$

TABLE 1

Orbifold	Dominant modulus T_d	ρ	ReT_d
$Z_3 \times Z_3$	T_1 or T_2 or T_3	-1	8.2
$Z_6 \times Z_3$	T_3	-1	8.2
$Z_4 \times Z_2$	T_1 or T_2	-0.75	10.2
$Z_4 \times Z_4$	T_1 or T_2 or T_3	-0.75	10.2
$Z_6 \times Z_2$ $Z_6 \times Z_3$	T_1 or T_2	-0.5, -1, -1.5, -2,-2.5 or -3	14.3, 8.2, 6.1, 5, 4.3 or 3.8
$Z_6 \times Z_6$	T_1 or T_2 or T_3	-0.5,-1, -1.5, -2,-2.5 or -3	14.3, 8.2, 6.1, 5, 4.3 or 3.8
$Z_6' \times Z_2$	T_1 or T_2 or T_3	-0.5, -1	14.3 or 8.2

TABLE 2

Orbifold	Dominant modulus T_d	ρ	ReT_d
$Z_6' \times Z_2$	T_1 or T_2 or T_3	18 values in the range $-\frac{1}{6}$ to $-\frac{5}{3}$	38 to 5.7
$Z_6 \times Z_2$	T_1 or T_2	38 values in the range $-\frac{1}{6}$ to $-\frac{41}{12}$	38 to 3.5
$Z_6 \times Z_3$		and two other values very close to 0	
$Z_6 \times Z_6$	T_1 or T_2 or T_3	38 values in the range $-\frac{1}{6}$ to $-\frac{41}{12}$	38 to 3.5
		and two other values very close to 0	

TABLE 3

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